**MATHEMATICS – 4 th semester**

**(All branches except Electrical, Electronics, Computer science, Information Technology and Applied Electronics)**

| CODE | COURSE NAME | CATEGORY | L | T | P | CREDIT |
|---|---|---|---|---|---|---|
| MAT 202 | PROBABILITY,STATISTICS AND NUMERICAL METHODS | BASIC SCIENCE COURSE | 3 | 1 | 0 | 4 |

**Preamble:** This course introduces students to the modern theory of probability and statistics, covering important models of random variables and techniques of parameter estimation and hypothesis testing. A brief course in numerical methods familiarises students with some basic numerical techniques for finding roots of equations, evaluationg definite integrals solving systems of linear equations, and solving ordinary differential equations which are especially useful when analytical solutions are hard to find.

**Prerequisite:** A basic course in one-variable and multi-variable calculus**.**

**Course Outcomes:** After the completion of the course the student will be able to

| | |
|---|---|
| **CO 1** | Understand the concept, properties and important models of discrete random variables and,using them, analyse suitable random phenomena. |
| **CO 2** | Understand the concept, properties and important models of continuous random variables and,using them, analyse suitable random phenomena. |
| **CO 3** | Perform statistical inferences concerning characteristics of a population based on attributes of samples drawn from the population |
| **CO 4** | Compute roots of equations, evaluate definite integrals and perform interpolation on given numerical data using standard numerical techniques |
| **CO 5** | Apply standard numerical techniques for solving systems of equations, fitting curves on given numerical data and solving ordinary differential equations. |

**Mapping of course outcomes with program outcomes**

| | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 | PO 10 | PO 11 | PO 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO 1 | 3 | 2 | 2 | 2 | 2 | | | | | 2 | | 1 |
| CO 2 | 3 | 2 | 2 | 2 | 2 | | | | | 2 | | 1 |
| CO 3 | 3 | 2 | 2 | 2 | 2 | | | | | 2 | | 1 |
| CO 4 | 3 | 2 | 2 | 2 | 2 | | | | | 2 | | 1 |
| CO 5 | 3 | 2 | 2 | 2 | 2 | | | | | 2 | | 1 |

**Assessment Pattern**

| Bloom's Category | Continuous Assessment Tests(%) | | End Semester Examination(%) |
|---|---|---|---|
| | 1 | 2 | |
| Remember | 10 | 10 | 10 |
| Understand | 30 | 30 | 30 |
| Apply | 30 | 30 | 30 |
| Analyse | 20 | 20 | 20 |
| Evaluate | 10 | 10 | 10 |
| Create | | | |

**End Semester Examination Pattern:** There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

**Course Level Assessment Questions**

**Course Outcome 1 (CO1):**

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.
2. An equipment consists of 5 componets each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the componets are operational, what is the probability that it functions properly?
3. X is a binomial random variable $B(n,p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?
4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

**Course Outcome 2 (CO2)**

1. What can you say about $P(X = a)$for any real number $a$when $X$is a (i) discrete random variable? (ii) continuous random variable?

2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twise the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter $\mu$ and Y following and exponential distribution with parameter $\lambda$. Find $P(X + Y \leqslant 1)$

**Course Outcome 3(CO3):**

1. In a random sample of 500 people selected from the population of a city 60 were found to be left-handed. Find a 95% confidence interval for the proportion of left-handed people in the city population.

2. What are the types of errors involved in statistical hypothesis testing. Explain the level of risks associated with each type of error.

3. A soft drink maker claims that a majority of adults prefer its leading beverage over that of its main competitor's. To test this claim 500 randomly selected people were given the two beverages in random order to taste. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Determine whether there is sufficient evidence, at the 5% level of significance, to support the soft drink maker's claim against the default that the population is evenly split in its preference.

4. A nutritionist is interested in whether two proposed diets, *diet A* and *diet B* work equally well in providing weight-loss for customers. In order to assess a difference between the two diets, she puts 50 customers on diet A and 60 other customers on diet B for two weeks. Those on the former had weight losses with an average of 11 pounds and a standard deviation of 3 pounds, while those on the latter lost an average of 8 pounds with a standard deviation of 2 pounds. Do the diets differ in terms of their weight loss?

**Course Outcome 4(CO4):**

1. Use Newton-Raphson method to find a real root of the equation $f(x) = e^{2x} - x - 6$correct to 4 decimal places.

2. Compare Newton's divided difference method and Langrange's method of interpolation.

3. Use Newton's forward interpolation formula to compute the approximate values of the function $f$ at $x = 0.25$ from the following table of values of $x$ and $f(x)$

| x | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| f(x) | 1.0000 | 1.0513 | 1.1052 | 1.1618 | 1.2214 |

4. Find a polynomial of degree 3 or less the graph of which passes thorugh the points (-1,3), (0,-4), (1,5) and (2,-6)

**Course Outcome 5 (CO5):**

1. Apply Gauss-Seidel method to solve the following system of equations
$$4x_1 - x_2 - x_3 = 3$$
$$-2x_1 + 6x_2 + x_3 = 9$$
$$-x_1 + x_2 + 7x_3 = -6$$

2. Using the method of least squares fit a straight line of the form $y = ax + b$ to the following set of ordered pairs $(x, y)$ :
(2,4), (3,5), (5,7), (7,10), (9,15)

3. Write the normal equations for fitting a curve of the form $y = a_0 + a_1x^2$ to a given set of pairs of data points.

4. Use Runge-Kutta method of fourth order to compute $y(0.25)$ and $y(0.5)$, given the initial value problem
$$y' = x + xy + y, y(0) = 1$$

**Syllabus**

**Module 1 (Discrete probability distributions)** **9 hours**

**(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)**

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation -multiple random variables.

**Module 2 (Continuous probability distributions)** **9 hours**

**(Text-1:*Relevant topics* from sections-4.1-4.4, 3.6, 5.1)**

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation-multiple random variables, i.i.d random variables and Central limit theorem (**without proof**).

**Module 3 (Statistical inference)** **9 hours**

**(Text-1**:***Relevant topics*** **from sections-5.4.,, 3.6, 5.1,7.2, 8.1, 8.3, 9.1-9.2,9.4)**

Population and samples, Sampling distribution of the mean and proportion (for large samples only), Confidence interval for single mean and single proportions(for large samples only). Test of hypotheses: Large sample test for single mean and single proportion, equality of means and equality of proportions of two populations, small sample t-tests for single mean of normal population, equality of means (**only pooled t-test, for independent samples from two normal populations with equal variance** )

**Module 4 (Numerical methods -I)** **9 hours**

**(Text 2-** ***Relevant topics*** **from sections 19.1, 19.2, 19.3, 19.5**)

Errors in numerical computation-round-off, truncation and relative error, Solution of equations – Newton-Raphson method and Regula-Falsi method. Interpolation-finite differences, Newton's forward and backward difference method, Newton's divided difference method and Lagrange's method. Numerical integration-Trapezoidal rule and Simpson's 1/3rd rule (**Proof or derivation of the formulae not required for any of the methods in this module**)

**Module 5 (Numerical methods -II)** **9 hours**

**(Text 2-** ***Relevant topics*** **from sections 20.3, 20.5, 21.1**)

Solution of linear systems-Gauss-Siedal and Jacobi iteration methods. Curve fitting-method of least squares, fitting staright lines and parabolas. Solution of ordinary differential equations-Euler and Classical Runge-Kutta method of second and fourth order, Adams-Moulton predictor-correction method (**Proof or derivation of the formulae not required for any of the methods in this module**)

**Text Books**

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, $8^{th}$ edition, Cengage, 2012
2. (Text-2) Erwin Kreyszig, *Advanced Engineering Mathematics,* 10 th Edition, John Wiley & Sons, 2016.

**Reference Books**

1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 ( Also available online at www.probabilitycourse.com )
2. Sheldon M. Ross, *Introduction to probability and statistics for engineers and*

*scientists*, 4th edition, Elsevier, 2009.
3. T. Veera Rajan, *Probability, Statistics and Random processes,* Tata McGraw-Hill, 2008
4. B.S. Grewal, *Higher Engineering Mathematics*, Khanna Publishers, 36 Edition, 2010.

**Assignments**

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

**Course Contents and Lecture Schedule**

| No | Topic | No. of Lectures |
|---|---|---|
| **1** | **Discrete Probability distributions** | **9 hours** |
| 1.1 | Discrete random variables and probability distributions, expected value, mean and variance (discrete) | 3 |
| 1.2 | Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial | 3 |
| 1.3 | Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values | 3 |
| **2** | **Continuous Probability distributions** | **9 hours** |
| 2.1 | Continuous random variables and probability distributions, expected value, mean and variance (continuous) | 2 |
| 2.2 | Uniform, exponential and normal distributions, mean and variance of these distributions | 4 |
| 2.3 | Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem. | 3 |
| **3** | **Statistical inference** | **9 hours** |
| 3.1 | Population and samples, Sampling distribution of single mean and single proportion( large samples) | 1 |
| 3.2 | Confidence interval for single mean and single proportions ( large samples) | 2 |
| 3.3 | Hypothesis testing basics, large sample test for single proportion, single proportion | 2 |
| 3.4 | Large sample test for equality of means and equality of proportions of two populations | 2 |

| | | |
|---|---|---|
| 3.5 | t-distribution and small sample t-test for single mean and pooled t-test for equality of means | 2 |
| **4** | **Numerical methods-I** | **9 hours** |
| 4.1 | Roots of equations- Newton-Raphson, regulafalsi methods | 2 |
| 4.2 | Interpolation-finite differences, Newton’s forward and backward formula, | 3 |
| 4.3 | Newton’s divided difference method, Lagrange’s method | 2 |
| 4.3 | Numerical integration-trapezoidal rule and Simpson’s 1/3-rd rule | 2 |
| **5** | **Numerical methods-II** | **9 hours** |
| 5.1 | Solution of linear systems-Gauss-Siedal method, Jacobi iteration method | 2 |
| 5.2 | Curve-fitting-fitting straight lines and parabolas to pairs of data points using method of least squares | 2 |
| 5.3 | Solution of ODE-Euler and Classical Runge-Kutta methods of second and fourth order | 4 |
| 5.4 | Adams-Moulton predictor-corrector methods | 1 |

**Model Question Paper**

**(2019 Scheme)**

**Reg No: .................** **Total Pages: 4**

**Name: .....................**

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

FOURTH SEMESTER B.TECH DEGREE EXAMINATION

(Month & year)

**Course Code: MAT**

**Course Name: PROBABILITY, STATISTICS AND NUMERICAL METHODS**

(Common to all branches except (i) Electrical and Electronics, (ii) Electronics and Communication, (iii) Applied Electronics and Instrumentation (iv) Computer Science and Engineering (v) Information Technology )

Max Marks :100 Duration : 3 Hours

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**PART A**

**(Answer *all* questions. Each question carries 3 marks)**

1. Suppose $X$ is binomial random variable with parameters $n = 100$ and $p = 0.02$. Find $P(X < 3)$ using Poisson approximation to $X$. (3)

2. The diameter of circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)

3. Find the mean and variance of the continuous random variable $X$ with probability density function (3)
$$f(x) = \begin{cases} 2x-4, & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

4. The random variable $X$ is exponentially distributed with mean 3. Find $P(X > t+3 | X > t)$ where $t$ is any positive real number. (3)

5. The 95% confidence interval for the mean mass (in grams) of tablets produced by a machine is $[0.56 \quad 0.57]$, as calculated from a random sample of 50 tablets. What do you understand from this statement? (3)

6. The mean volume of liquid in bottles of lemonade should be at least 2 litres. A sample of bottles is taken in order to test whether the mean volume has fallen below 2 litres. Give a null and alternate hypothesis for this test and specify whether the test would be one-tailed or two-tailed. (3)

7. Find all the first and second order forward and backward differences of $y$ for the following set of $(x, y)$ values: $(0.5, 1.13)$, $(0.6, 1.19)$, $(0.7, 1.26)$, $(0.8, 1.34)$ (3)

8. The following table gives the values of a function $f(x)$ for certain values of $x$. (3)

| $x$ | 0 | 0.25 | 0.50 | 0.75 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.9412 | 0.8 | 0.64 | 0.5 |

Evaluate $\int_0^1 f(x)dx$ using trapezoidal rule.

9. Explain the principle of least squares for determining a line of best fit to a given data (3)

10. Given the initial value problem $y' = y + x, \quad y(0) = 0$, find $y(0.1)$ and $y(0.2)$ using Euler method. (3)

**PART B**
**(Answer one question from each module)**
**MODULE 1**

11. (a) The probability mass function of a discrete random variable is $p(x) = kx, x = 1, 2, 3$ where $k$ is a positive constant. Find (i)the value of $k$ (ii) $P(X \leq 2)$ (iii) $E[X]$ and (iv) var$(1 - X)$. (7)

(b) Find the mean and variance of a binomial random variable (7)

**OR**

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. what is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

(b) Two fair dice are rolled. Let $X$ denote the number on the first die and $Y = 0$ or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of $X$ and $Y$, (ii) the marginal distributions. (iii) Are $X$ and $Y$ independent ? (7)

**MODULE 2**

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

(b) A continuous random variable $X$ is uniformly distributed with mean 1 and variance 4/3. Find $P(X < 0)$ (7)

**OR**

14. (a) The joint density function of random variables $X$ and $Y$ is given by (7)

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X + Y \leq 1)$. Are $X$ and $Y$ independent? Justify.

(b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

**MODULE 3**

15. (a) The mean blood pressure of 100 randomly selected persons from a target population is 127.3 units. Find a 95% confidence interval for the mean blood pressure of the population. (7)

(b) The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisfied. Based on these findings, do you think that the CEO is making a false claim of high satisfaction levels among his customers? Use a 0.05 level of significance. (7)

**OR**

16. (a) A magazine reported the results of a telephone poll of 800 adult citizens of a country. The question posed was: "Should the tax on cigarettes be raised to pay for health care reform?" The results of the survey were: Out of the 800 persons surveyed, 605 were non-smokers out of which 351 answered "yes" and the rest "no". Out of the remaining 195, who were smokers, 41 answered "yes" and the remaining "no". Is there sufficient evidence, at the 0.05 significance level, to conclude that the two populations smokers and non-smokers differ significantly with respect to their opinions? (7)

(b) Two types of cars are compared for acceleration rate. 40 test runs are recorded for each car and the results for the mean elapsed time recorded below: (7)

| | Sample mean | Sample standard deviation |
|---|---|---|
| Car A | 7.4 | 1.5 |
| Car B | 7.1 | 1.8 |

determine if there is a difference in the mean elapsed times of the two car models at 95% confidence level.

**MODULE 4**

17. (a) Use Newton-Raphson method to find a non-zero solution of $x = 2\sin x$. Start with $x_0 = 1$ (7)

(b) Using Lagrange's interpolating polynomial estimate $f(1.5)$ for the following data (7)

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y = f(x)$ | 0 | 0.9826 | 0.6299 | 0.5532 |

**OR**

18. (a) Consider the data given in the following table (7)

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 1.0000 | 1.0513 | 1.1052 | 1.1618 | 1.2214 |

Estimate the value of $f(1.80)$ using newton's backward interpolation formula.

(b) Evaluate $\int_0^1 e^{-x^2/2}dx$ using Simpson's one-third rule, dividing the interval [0, 1] into 8 subintervals (7)

**MODULE 5**

19. (a) Using Gauss-Seidel method, solve the following system of equations (7)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) The table below gives the estimated population of a country (in millions) for during 1980-1995 (7)

| year | 1980 | 1985 | 1990 | 1995 |
|---|---|---|---|---|
| population | 227 | 237 | 249 | 262 |

Plot a graph of this data and fit an appropriate curve to the data using the method of least squares. Hence predict the population for the year 2010.

**OR**

20. (a) Use Runge-Kutta method of fourth order to find $y(0.2)$ given the initial value problem (7)

$$\frac{dy}{dx} = \frac{xy}{1+x^2}, \quad y(0) = 1$$

Take step-size, $h = 0.1$.

(b) Solve the initial value problem (7)

$$\frac{dy}{dx} = x + y, \quad y(0) = 0,$$

in the interval $0 \leq x \leq 1$, taking step-size $h = 0.2$. Calculate $y(0.2), y(0.4)$ and $y(0.6)$ using Runge-Kutta second order method, and $y(0.8)$ and $y(1.0)$ using Adam-Moulton predictor-corrector method.

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