MATHEMATICS – Third Semester B. Tech

(For all branches except Computer Science and Information Technology)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT201	PARTIAL DIFFERENTIAL EQUATIONS AND COMPLEX ANALYSIS	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces basic ideas of partial differential equations which are widely used in the modelling and analysis of a wide range of physical phenomena and has got application across all branches of engineering. To understand the basic theory of functions of a complex variable, residue integration and conformal transformation.

Prerequisite: A basic course in partial differentiation and complex numbers**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept and the solution of partial differential equation.
CO 2	Analyse and solve one dimensional wave equation and heat equation.
CO 3	Understand complex functions, its continuity differentiability with the use of Cauchy-Riemann equations.
CO 4	Evaluate complex integrals using Cauchy's integral theorem and Cauchy's integral formula, understand the series expansion of analytic function
CO 5	Understand the series expansion of complex function about a singularity and Apply residue theorem to compute several kinds of real integrals.

Mapping of course outcomes with program outcomes

PO's	Broad area
PO 1	Engineering Knowledge
PO 2	Problem Analysis
PO 3	Design/Development of solutions
PO 4	Conduct investigations of complex problems
PO 5	Modern tool usage
PO 6	The Engineer and Society
PO 7	Environment and Sustainability
PO 8	Ethics
PO 9	Individual and team work

PO 10	Communication
PO 11	Project Management and Finance
PO 12	Life long learning

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1				2		2
CO 2	3	3	3	3	2	1				2		2
CO 3	3	3	3	3	2	1				2		2
CO 4	3	3	3	3	2	1				2		2
CO 5	3	3	3	3	2	1				2		2

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions.

Course Outcome 1 (CO1):

1. Form the partial differential equation given $z = xf(x) + ye^2$
2. What is the difference between complete integral and singular integral of a partial differential equation
3. Solve $3z = xp + yq$
4. Solve $(p^2 + q^2)y = qz$
5. Solve $u_x - 2u_t = u$ by the method of separation of variables

Course Outcome 2 (CO2):

1. Write any three assumptions in deriving one dimensional wave equations
2. Derive one Dimensional heat equation
3. Obtain a general solution for the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial t^2}$
4. A tightly stretched flexible string has it's ends fixed at $x = 0$and $x = l$. At $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$ where μ is a constant
5. Find the temperature $u(x, t)$ in a bar which is perfectly insulated laterally whose ends are kept at $0°C$ and whose initial temperature (in degree Celsius) is $f(x) = x(10 - x)$ given that it's length is 10 cm and specific heat is 0.056cal/gram deg

Course Outcome 3(CO3):

1. Separate the real and imaginary parts of $f(z) = \frac{1}{1+z}$
2. Check whether the function $f(z) = \frac{Re(z^2)}{|z|}$ is continuous at $z = 0$ given $f(0) = 0$
3. Determine a and b so that function $u = e^{-\pi x} cosay$ is harmonic. Find it's harmonic conjugate.
4. Find the fixed points of $w = \frac{i}{2z-1}$
5. Find the image of $|z| \leq \frac{1}{2}$, $-\frac{\pi}{8} < argz < \frac{\pi}{8}$under $w = z^2$

Course Outcome 4(CO4):

1. Find the value of $\int_C \ exp(z^2)dz$ where C is $|z| = 1$
2. Integrate the function $\int_C \frac{sinz}{z+4iz} dz$ where C is $|z - 4 - 2i| = 6.5$
3. Evaluate $\int_C \frac{e^z}{\left(z-\frac{\pi}{4}\right)^3} dz$ where C is $|z| = 1$
4. Find the Maclaurin series expansion of $f(z) = \frac{i}{1-z}$ and state the region of convergence.
5. Find the image of $|z| = 2$ under the mapping $w = z + \frac{1}{z}$

Course Outcome 5 (CO5):

1. Determine the singularity of $exp\left(\frac{1}{z}\right)$
2. Find the Laurent series of $\frac{1}{z^2(z-i)}$ about $z = i$
3. Find the residues of $f(z) = \frac{50z}{z^3 + 2z^2 - 7z + 4}$
4. Evaluate $\int_C \ tan2\pi z \, dz$ where C is $|z - 0.2| = 0.2$
5. Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$

Syllabus

Module 1 (Partial Differential Equations) (8 hours)

(Text 1-Relevant portions of sections 17.1, 17.2, 17.3, 17.4, 17.5, 17.7, 18.1, 18.2)

Partial differential equations, Formation of partial differential equations –elimination of arbitrary constants-elimination of arbitrary functions, Solutions of a partial differential equations, Equations solvable by direct integration, Linear equations of the first order-Lagrange's linear equation, Non-linear equations of the first order -Charpit's method, Solution of equation by method of separation of variables.

Module 2 (Applications of Partial Differential Equations) (10 hours)

(Text 1-Relevant portions of sections 18.3,18.4, 18.5)

One dimensional wave equation- vibrations of a stretched string, derivation, solution of the wave equation using method of separation of variables, D'Alembert's solution of the wave equation, One dimensional heat equation, derivation, solution of the heat equation

Module 3 (Complex Variable – Differentiation) (9 hours)

(Text 2: Relevant portions of sections13.3, 13.4, 17.1, 17.2 , 17.4)

Complex function, limit, continuity, derivative, analytic functions, Cauchy-Riemann equations, harmonic functions, finding harmonic conjugate, Conformal mappings- mappings $w = z^2$,, $w = e^z$,. Linear fractional transformation $w = \frac{1}{z}$, fixed points, Transformation $w = sinz$

(From sections 17.1, 17.2 and 17.4 only mappings $w = z^2$,, $w = e^z$, $w = \frac{1}{z}$, $w = sinz$ and problems based on these transformation need to be discussed)

Module 4 (Complex Variable – Integration) (9 hours)

(Text 2- Relevant topics from sections14.1, 14.2, 14.3, 14.4,15.4**)**

Complex integration, Line integrals in the complex plane, Basic properties, First evaluation method-indefinite integration and substitution of limit, second evaluation method-use of a representation of a path, Contour integrals, Cauchy integral theorem (without proof) on simply connected domain,Cauchy integral theorem (without proof) on multiply connected domain Cauchy Integral formula (without proof), Cauchy Integral formula for derivatives of an analytic function, Taylor's series and Maclaurin series.,

Module 5 (Complex Variable – Residue Integration) (**9 hours**)

(Text 2- Relevant topics from sections16.1, 16.2, 16.3, 16.4 **)**

Laurent's series(without proof), zeros of analytic functions, singularities, poles, removable singularities, essential singularities, Residues, Cauchy Residue theorem (without proof), Evaluation of definite integral using residue theorem, Residue integration of real integrals – integrals of rational functions of $cos\theta$ and $sin\theta$, integrals of improper integrals of the form

$\int_{-\infty}^{\infty} f(x)\,dx$ with no poles on the real axis. ($\int_{A}^{B} f(x)\,dx$ whose integrand become infinite at a point in the interval of integration is excluded from the syllabus),

Textbooks:

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 44th Edition, 2018.

2. Erwin Kreyszig, Advanced Engineering Mathematics, 10th Edition, John Wiley & Sons, 2016.

References:

1. Peter V. O'Neil, Advanced Engineering Mathematics, Cengage, 7th Edition, 2012

Assignments

Assignment: Assignment must include applications of the above theory in the concerned engineering branches

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Partial Differential Equations**	
1.1	Partial differential equations, Formation of partial differential equations –elimination of arbitrary constants-elimination of arbitrary functions, Solutions of a partial differential equations, Equations solvable by direct integration,	3
1.2	Linear equations of the first order- Lagrange's linear equation, Non-linear equations of the first order - Charpit's method	3
1.3	Boundary value problems, Method of separation of variables.	2
2	**Applications of Partial Differential Equations**	
2.1	One dimensional wave equation- vibrations of a stretched string, derivation,	1
2.2	Solution of wave equation using method of separation of variables, Fourier series solution of boundary value problems involving wave equation, D'Alembert's solution of the wave equation	4
2.3	One dimensional heat equation, derivation,	1
2.4	Solution of the heat equation, using method of separation of variables, Fourier series solutions of boundary value problems involving heat equation	4

3	**Complex Variable – Differentiation**	
3.1	Complex function, limit, continuity, derivative, analytic functions, Cauchy-Riemann equations,	4
3.2	harmonic functions, finding harmonic conjugate,	2
3.3	Conformal mappings- mappingsof $w = z^2,, w = e^z, w = \frac{1}{z}, w = sinz$.	3
4	**Complex Variable – Integration**	
4.1	Complex integration, Line integrals in the complex plane, Basic properties, First evaluation method, second evaluation method, use of representation of a path	4
4.2	Contour integrals, Cauchy integral theorem (**without proof**) on simply connected domain, on multiply connected domain(**without proof**) .Cauchy Integral formula (**without proof**),	2
4.3	Cauchy Integral formula for derivatives of an analytic function,	2
4.3	Taylor's series and Maclaurin series.	1
5	**Complex Variable – Residue Integration**	
5.1	Laurent's series(**without proof**)	2
5.2	zeros of analytic functions, singularities, poles, removable singularities, essential singularities, Residues,	2
5.3	Cauchy Residue theorem (**without proof**), Evaluation of definite integral using residue theorem	2
5.4	Residue integration of real integrals – integrals of rational functions of $cos\theta$and $sin\theta$, integrals of improper integrals of the form $\int_{-\infty}^{\infty} f(x)dx$with no poles on the real axis. ($\int_A^B f(x)dx$**whose integrand become infinite at a point in the interval of integration is excluded from the syllabus**),	3

Model Question Paper

(For all branches except Computer Science and Information Technology)

(2019 Scheme)

Reg No: Name:

APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH. DEGREE EXAMINATION

(MONTH & YEAR)

Course Code:

Course Name: PARTIAL DIFFERENTIAL EQUATIONS AND COMPLEX ANALYSIS

MAX.MARKS: 100 DURATION: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1. Derive a partial differential equation from the relation $z = f(x + at) + g(x - at)$
2. Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$
3. State any three assumptions in deriving the one dimensional wave equation
4. What are the possible solutions of one-dimensional heat equation?
5. If $f(z) = u + iv$ is analytic, then show that u and v are harmonic functions.
6. Check whether $f(z) = \bar{z}$ is analytic or not.
7. Evaluate $\int_c tan\, z\, dz$ where c is the unit circle.
8. Find the Taylor's series of $f(z) = \frac{1}{z}$ about $z = 2$.
9. What type of singularity have the function $f(z) = \frac{1}{\cos z - \sin z}$
10. Find the residue of $\frac{e^z}{z^3}$ at its pole.

PART B

Answer any one full question from each module, each question carries 14 marks.

Module-I

11. (a) Solve $x(y - z)p + y(z - x)q = z(x - y)$
    (b) Use Charpit's methods to solve $q + xp = p^2$
12. (a) Find the differential equation of all spheres of fixed radius having their centers in the xy-plane.

(b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$.

Module – II

13. (a) Derive the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with zero boundary conditions and with initial conditions $u(x,0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$.

(b) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is $u(x,0) = \begin{cases} x, 0 \le x \le 50 \\ 100 - x, 50 \le x \le 100 \end{cases}$. Find the temperature $u(x,t)$ at any time.

14. (a) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0\, sin^3\left({\pi x}/{l}\right)$. Find the displacement of the string at any time.

(b) An insulated rod of length l has its ends A and B are maintained at $0^0 c$ and $100^0 C$ respectively under steady state condition prevails. If the temperature at B is suddenly reduced to $0^0 c$ and maintained at $0^0 c$, Find the temperature at a distance x from A at time t.

Module-III

15. (a) Show that $f(z) = e^z$ is analytic for all z. Find its derivative.

(b) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$

16. (a) Prove that the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic everywhere. Find its harmonic conjugate.

(b) Find the image of the infinite stripe $0 \le y \le \pi$ under the transformation $w = e^z$

Module-IV

17. (a) Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$, along the real axis to 2 and then vertically to $2 + i$

(b) Using Cauchy's integral formula evaluate $\int_c \frac{5z+7}{z^2+2z-3}\, dz$, where c is $|z - 2| = 2$

18. (a) Evaluate $\int_c \frac{sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where C is $|z| = 1$.

(b) Expand $\frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$

Module- V

19. (a) Expand $f(z) = \frac{z^2-1}{z^2-5z+6}$ in $2 < |z| < 3$ as a Laurent's series.

(b) Using contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{2 + cos\,\theta}$

20. (a) Use residue theorem to evaluate $\int_c \frac{cos\, h\, \pi z}{z^2+4} dz$ where are C is $|z| = 3$.

(b) Apply calculus of residues to evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3} dx$.