

CET394	FINITE ELEMENT METHODS	CATEGORY	L	T	P	CREDIT	Year of Introduction
		VAC	3	1	0	4	2019

Preamble: This course provides the fundamental concepts of finite element method and its applications in structural engineering. As a natural development from matrix analysis of structures learnt earlier, the student is encouraged to appreciate the versatility of this method across various domains, also as the basis of many structural analysis softwares. This course introduces the basic mathematical concepts of the method and its application to simple analysis problems.

Prerequisite: CET302 Structural Analysis II

Course Outcomes: After the completion of the course the student will be able to

Course Outcome	Description of Course Outcome	Prescribed learning level
CO1	Understand the basic features of boundary value problems and methods to solve them.	Remembering, Understanding
CO2	Understand the fundamental concept of the finite element method and develop the ability to generate the governing FE equations for systems governed by partial differential equations.	Understanding, Applying
CO3	Get familiar with the basic element types and shape functions so as to identify and choose suitable elements to solve a particular problem.	Analysing, Applying
CO4	Understand the concept of isoparametric elements and apply it for problems in structural engineering.	Understanding, Applying
CO5	Apply numerical integration procedures as a tool to solve mathematical models in FEM.	Understanding, Applying

Mapping of course outcomes with program outcomes (Minimum requirement)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	2	1	-	-	-	-	-	-	-	-
CO2	3	3	2	1	-	-	-	-	-	-	-	-
CO3	3	3	1	1	-	-	-	-	-	-	-	-
CO4	3	3	1	-	-	-	-	-	-	-	-	-
CO5	3	3	1	1	-	-	-	-	-	-	-	-

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember	05	05	10
Understand	10	10	20
Apply	20	20	40
Analyse	15	15	30
Evaluate			
Create			

Mark distribution

Total Marks	CIE	ESE	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

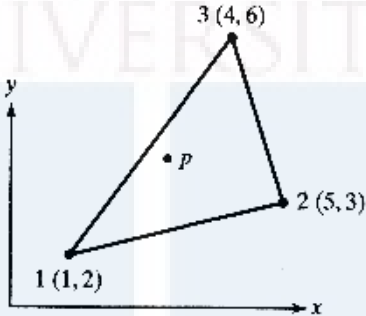

Attendance : 10 marks
 Continuous Assessment Test (2 numbers) : 25 marks
 Assignment/Quiz/Course project : 15 marks

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question carries 14 marks and can have maximum 2 sub-divisions.

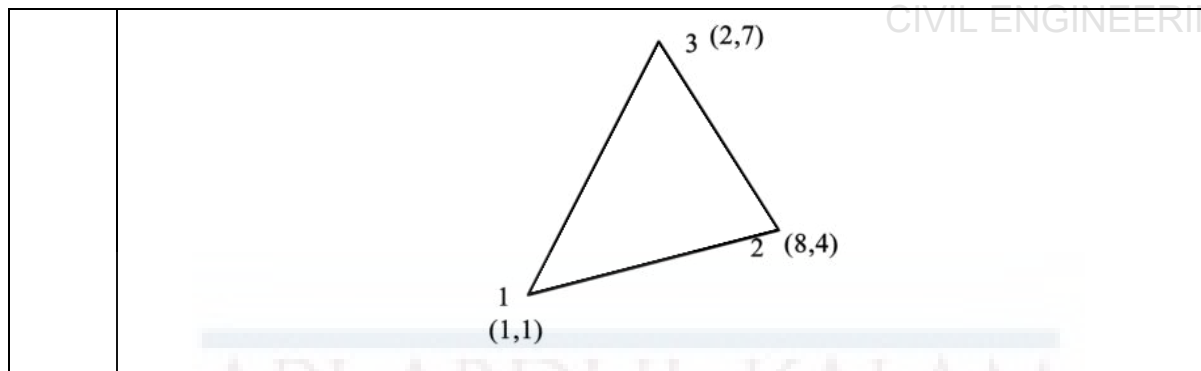
Course Level Assessment Questions

CO1:	Understand the basic features of boundary value problems and methods to solve them.
1.	What are boundary value problems? What are the physical and mathematical significances of boundary conditions in structural mechanics problems?
2.	Using the Galerkin method obtain an approximate solution to the following boundary value problem. $u''(x) + u(x) + x = 0 \quad 0 < x < 1$ $u(0) = 0 \quad u(1) = 0$ <p>(a) Assume a quadratic polynomial as a trial solution. (b) Assume a cubic polynomial as a trial solution.</p>
3.	Find a one-parameter approximate solution of the nonlinear equation $-2u \frac{d^2 u}{dx^2} + \left(\frac{du}{dx} \right)^2 = 4 \quad \text{for } 0 < x < 1,$ subject to the boundary conditions $u(0) = 1$ and $u(1) = 0$, and compare it with the exact solution $u = 1 - x^2$. Use the least-squares method.

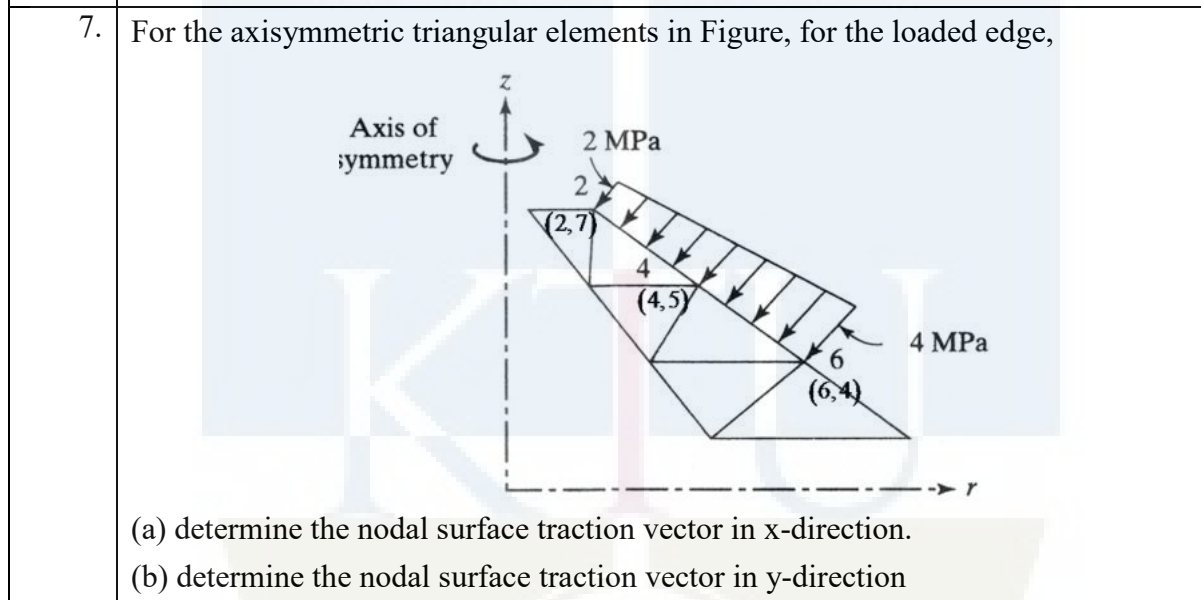
CO2:	Understand the fundamental concept of the finite element method and develop the ability to generate the governing FE equations for systems governed by partial differential equations.
1.	Derive the governing differential equation of a uniform bar subjected to axial vibrations.
2.	What are field variables and forcing vectors in finite element analysis? Give examples from various applications.
3.	Derive the element stiffness equations for an axial deformation problem, using variational approach.
4.	(a) Obtain the weak form of the following boundary value problem. $x^2 \frac{d^2 u}{dx^2} + 2x \frac{du}{dx} - xu + 4 = 0 \quad 1 < x < 3$ $u(1) = 1 \quad \frac{du(3)}{dx} - 2u(3) = 2$ <p>(b) With the weak form obtained in (a), use Rayleigh-Ritz method to obtain an approximate solution of the above BVP. Use a linear polynomial trial solution.</p>

CO3:	Get familiar with the basic element types and shape functions so as to identify and choose suitable elements to solve a particular problem.
1.	What are shape functions? What are their advantages in finite element analysis?
2.	Obtain the shape functions for a 4-noded bar element using Lagrange polynomials.
3.	Write the elasticity relations for axisymmetric elements.
4.	<p>For the CST element in figure, x-coordinate at P is 3 and N2 is 0.4 at P. Determine:</p> <p>(a) the y-coordinate at P</p> <p>(b) N1 and N3 at P.</p> 
5.	<p>Get the explicit shape functions for the rectangular element shown in Figure 3, using Lagrange formulae.</p> 

CO4:	Understand the concept of isoparametric elements and apply it for problems in structural engineering.
1.	Find the axial deformation of a mild steel square bar of side 3cm and length 2m, using two linear isoparametric axial elements.
2.	Derive the shape functions for an isoparametric Constant Strain Triangle element.
3.	Find the isoparametric mapping for the CST element shown.



4. What are the advantages of coordinate mapping?
5. What are superparametric, subparametric and isoparametric elements?
6. Illustrate the influence of node numbering on Jacobian, by using a linear triangular isoparametric element.



CO5:	Apply numerical integration procedures as a tool to solve mathematical models in FEM.
1.	Evaluate the following integrals using Gauss quadrature: $(a) \ I = \int_{0.2}^{0.8} e^{-2x} \tan x dx \quad (b) \ I = \int_{-2}^2 \frac{dx}{1+x^2}$ $(c) \ I = \int_{-1}^1 \int_{-1}^1 (t^3 + s^2) ds dt \quad (d) \ I = \int_{-1}^1 \int_{-1}^1 x \sin(x + y^2) dx dy$
2.	What are the essential features of numerical integration using Gauss quadrature?
3.	Obtain the two-point Gauss quadrature points and weights from first principles
4.	How to determine the number of Gauss points to evaluate an integral exactly?

MODULE I – 9 hrs.

Introduction - Boundary value problems; Introduction to approximate numerical solutions for solving differential equations.

MODULE II – 9 hrs.

Formulation techniques: Element equations using variational approach- Element equations using weighted residual approach - the axial element example.

MODULE III – 9 hrs.

Basic elements: Interpolation and shape functions – convergence requirements; CST, LST, bilinear rectangular elements, solid elements.

MODULE IV – 9 hrs.

Isoparametric Formulation: coordinate mapping - One dimensional bar element; Two dimensional isoparametric elements - CST, LST, bilinear quadrilateral elements - Plain stress, plain strain problems.

MODULE V – 9 hrs.

Development of stiffness matrix for *beam elements*; Introduction to *higher order* elements; Introduction to *axisymmetric* elements.

Numerical Integration: Gauss quadrature

Text Books:

1. Desai, C.S., Elementary Finite Element Method, Prentice Hall of India.
2. Chandrupatla, T.R., and Belegundu, A.D., Introduction to Finite Elements in Engineering, Prentice Hall of India.

References:

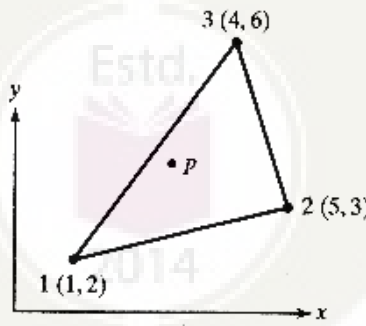
1. Cook, R.D., et al, Concepts and Applications of Finite Element Analysis, John Wiley.
2. Bathe, K.J., Finite Element Procedures in Engineering Analysis, Prentice Hall of India.
3. Gallagher, R.H., Finite Element Analysis: Fundamentals, Prentice Hall Inc.
4. Rajasekaran, S., Finite Element Analysis in Engineering Design, Wheeler Pub.
5. Krishnamoorthy, C.S., Finite Element Analysis Theory and Programming, Tata McGraw Hill.
6. Zienkiewicz, O.C., and Taylor, R.L., The Finite Element Method, Vol. I and II, McGraw Hill.
7. Bhatti, Asghar, Fundamental Finite Element Analysis and Applications: with Mathematica and Matlab Computations


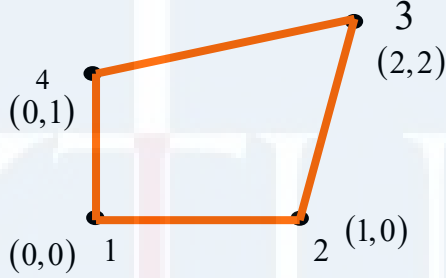
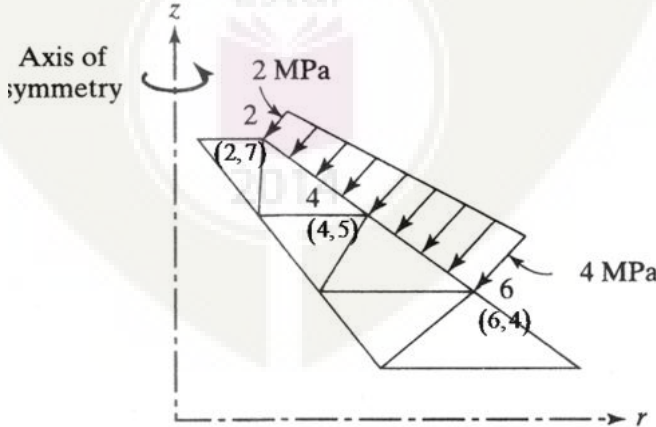
Lecture Plan –Structural Analysis II

CIVIL ENGINEERING

Module	Topic	Course Outcomes addressed	No. of Lectures
1	Module I: Total lecture hours: 9		
1.1	General introduction – brief review of matrix methods, applications and versatility of FEM	CO1	1
1.2	Introduction to Boundary value problems; approximate numerical solutions for solving differential equations - Least squares method	CO1	3
1.3	Collocation method, Galerkin method - examples	CO1	5
2	Module II: Total lecture hours: 9		
2.1	Formulation techniques: Variational approach and weighted residual approach – initial concepts and differences	CO2	1
	Element equations using variational approach		3
2.2	Element equations using weighted residual approach	CO2	3
2.3	The axial element example in detail	CO2, CO3	2
3	Module III: Total lecture hours: 9		
3.1	Basic elements: Interpolation and shape functions	CO3	2
3.2	Convergence requirements; CST element	CO3	3
3.3	LST, bilinear rectangular elements, solid elements.	CO3	4
4	Module IV: Total lecture hours: 9		
4.1	Isoparametric Formulation: coordinate mapping - One dimensional bar element	CO4	2
4.2	Two dimensional isoparametric elements – CST element	CO4	3
4.3	LST, bilinear quadrilateral elements - Plain stress, plain strain problems.	CO4	4
5	Module V: Total lecture hours: 9		
5.1	Development of stiffness matrix for beam elements	CO3, CO4	2
5.2	Introduction to higher order elements	CO3, CO4	2
5.3	Introduction to axisymmetric elements.	CO3, CO4	2
5.4	Numerical Integration: Gauss quadrature	CO5	3

Reg.No.: _____		Name: _____	
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY SIXTH SEMESTER B.TECH DEGREE EXAMINATION			
Course Code: CET394			
Course Name: FINITE ELEMENT METHODS			
Max. Marks: 100		Duration: 3 Hours	
PART A			
<i>Answer all questions; each question carries 3 marks.</i>			
1.	a)	What are field variables and forcing vectors in finite element analysis? Give examples from various applications.	
	b)	What are boundary value problems? What are the physical and mathematical significances of boundary conditions in structural mechanics problems?	
	c)	List the essential properties of shape functions.	
	d)	Briefly explain the essential features of weighted residual methods to solve partial differential equations.	
	e)	Write down the brief general procedure in finite element analysis.	
	f)	What are shape functions? What are their advantages in finite element analysis?	
	g)	What are the advantages of coordinate mapping?	
	h)	What are superparametric, subparametric and isoparametric elements?	
	i)	What are axisymmetric elements? Explain.	
	j)	How to determine the number of Gauss points to evaluate an integral exactly?	
(10×3 marks = 30 marks)			
PART B			
<i>Answer one full question from each module; each full question carries 14 marks.</i>			
Module I			
2.	Using the Galerkin method obtain an approximate solution to the following boundary value problem. $u''(x) + u(x) + x = 0 \quad 0 < x < 1$ $u(0) = 0 \quad u(1) = 0$ (a) Assume a quadratic polynomial as a trial solution. (b) Assume a cubic polynomial as a trial solution.		
(2×7=14 marks)			
3.	Find a one-parameter approximate solution of the nonlinear equation		

	$-2u \frac{d^2 u}{dx^2} + \left(\frac{du}{dx} \right)^2 = 4 \quad \text{for } 0 < x < 1,$ <p>subject to the boundary conditions $u(0) = 1$ and $u(1) = 0$, and compare it with the exact solution $u = 1 - x^2$. Use the least-squares method.</p> <p style="text-align: right;">(14 marks)</p>
Module II	
4.	Derive the element stiffness equations for an axial deformation problem, using variational approach. (14 marks)
5.	<p>(a) Obtain the weak form of the following boundary value problem.</p> $x^2 \frac{d^2 u}{dx^2} + 2x \frac{du}{dx} - xu + 4 = 0 \quad 1 < x < 3$ $u(1) = 1 \quad \frac{du(3)}{dx} - 2u(3) = 2$ <p>(b) With the weak form obtained in (a), use Rayleigh-Ritz method to obtain an approximate solution of the above BVP. Use a linear polynomial trial solution.</p> <p style="text-align: right;">(2×7=14 marks)</p>
Module III	
6.	<p>For the CST element in figure, x-coordinate at P is 3 and N2 is 0.4 at P. Determine:</p> <p>(a) the y-coordinate at P</p> <p>(b) N1 and N3 at P.</p> <div style="text-align: center;">  </div> <p style="text-align: right;">(2×7=14 marks)</p>
7.	Get the explicit shape functions for the rectangular element shown in Figure 3, using Lagrange formulae.

	 <p style="text-align: right;">(14 marks)</p>
Module IV	
8.	<p>Illustrate the influence of node numbering on Jacobian, by using a linear triangular isoparametric element.</p> <p style="text-align: right;">(14 marks)</p>
9.	<p>Get the explicit isoparametric shape functions for the quadrilateral element shown in Figure 4. Check the validity of isoparametric mapping.</p>  <p style="text-align: right;">(14 marks)</p>
Module V	
10.	<p>For the axisymmetric triangular elements in Figure, for the loaded edge,</p>  <p>(a) determine the nodal surface traction vector in x-direction.</p> <p>(b) determine the nodal surface traction vector in y-direction.</p> <p style="text-align: right;">(2×7=14 marks)</p>

11. Evaluate the following integrals using two-point Gauss quadrature:

$$(a) \ I = \int_1^2 \int_4^6 xy e^{(x^2+y^2)} dx dy \quad (b) \ I = \int_{-2}^2 \frac{dx}{1+x^2}$$

(2×7=14 marks)

